**Enrollment No:** _____ **Exam Seat No:**_____

# C. U. SHAH UNIVERSITY
## Winter Examination-2021

**Subject Name : Problem Solving-II**

**Subject Code : 5SC03PRS1** **Branch: M.Sc. (Mathematics)**

**Semester: 3** **Date: 17/12/2021** **Time: 02:30 To 05:30** **Marks: 70**

**Instructions:**

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

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## SECTION – I

**Q-1** **Attempt the Following questions.** **[07]**

**a.** Show that the set $\{1, -1, i, -i\}$ is an abelian group under usual multiplication. **(02)**

**b.** Define Monoid.Give two examples of Monoid which are not Groups. **(02)**

**c.** Classify the region in which the equation $4y^2r + x^2t = 9$ is elliptic. **(02)**

**d.** True or False: $\mathbf{Z} \times \mathbf{Z}$is a cyclic group. **(01)**

**Q-2** **Attempt all questions** **[14]**

**a.** Show that a finite group $G$ is cyclic group of order if and only if it has no proper subgroups. **(05)**

**b.** Let $G$ be a finite abelian group of order $n$ .When the map $x \to x^m$ be an automorphism ? Justify. **(05)**

**c.** Using Runge Kutta's method find $y(0.1)$ provided $\frac{dy}{dx} = x^2 - y$ with $y(0) = 1$. (Take $h = 0.1$). **(04)**

**OR**

**Q-2**

**a.** Let $o(G) = pq$ , $p$ and $q$ are prime numbers with $p < q$.If $p \nmid q - 1$ then show that $G$ is cyclic. **(06)**

**b.** Find the number of irreducible monic quadratic polynomial in $\mathbf{Z_P}[X]$ , where p is prime. **(05)**

**c.** Find isomorphic group to $U(720)$. **(03)**

**Q-3** **Attempt all questions.** **[14]**

**a.** Find complete integral of $z^2 = pqxy$ using Charpit's method. **(06)**

**b.** Evaluate : (i) $\Delta^2 \left(\frac{5x+12}{x^2+5x+6}\right)$ **(05)**
(ii) $\Delta(e^{ax} \log bx)$

**c.** Show that the polynomial $x^{p^n} - x \in Z_p[x]$ can't have a root with multiplicity greater than 1. **(03)**

**OR**

**Q-3**

**a.** Show that any group of order 20449 is abelian. **(06)**

**b.** Evaluate $f(8)$ using Newton's Divided difference formula from the following table: **(05)**

| X | 4 | 5 | 7 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|
| Y | 48 | 100 | 294 | 900 | 1210 | 2028 |

**c.** Does there exist a group G with $o\left(G/_{Z(G)}\right) = 97$ ?Justify. **(03)**

## SECTION – II

**Q-4** **Attempt the Following questions.** **[07]**

**a** Write $\beta^{99}$ in disjoint cycle form ,where $\beta = (1\ 2\ 3)(1\ 4\ 5)$. **(02)**

**b** Generate a field of order 9. **(02)**

**c** How many different binary operationscan be defined on the set $\{a, 1\}$ ? **(01)**

**d** Find $\Delta\, sinx$. **(01)**

**e** True or False:$\boldsymbol{Z_3 \times Z_6 \cong Z_{18}}$ **(01)**

**Q-5** **Attempt all questions** **[14]**

**a.** Solve the following P.D.E. :$(D - 2D')(D - 3D' + 2)z = e^{2x+y}(1 + xy)$ **(06)**

**b.** Using Picard's method find $y(0.1), y(0.2)$ given $y' = 1 + xy$ , $y(0) = 1$. **(05)**

**c.** Find all automorphism of $\boldsymbol{Z_n}$ . $(n \in \boldsymbol{N})$ . **(03)**

**OR**

**Q-5**

**a.** Use Lagrange's Inverse Interpolation Formula to find $x$ when $f(x) = 14$ given $f(0) = 16.35$ , $f(5) = 14.88$ , $f(10) = 13.59$ and $f(15) = 12.46$ **(06)**

**b.** With proper justification prove or disprove :If $G$ is a group of order $pq$ then $G$ has at least one subgroup having order $p$ ,where $p, q$ are primes numbers and $p > q$. Also ,state the result you use. **(06)**

**c.** Find order of any four elements in $U(15)$. **(02)**

**Q-6** **Attempt all questions** **[14]**

**a.** Solve the following system of linear equations using Gauss Seidel method: $28x + 4y - z = 32$ , $x + 3y + 10z = 24$ , $2x + 17y + 4z = 35$ . **(06)**

**b.** Solve the Heat Equation $\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial y^2} = \frac{1}{k}\frac{\partial \varphi}{\partial t}$ by the method of separation of variables and show that the solution is of the form $\varphi(x, y, t) = e^{\pm i(nx+my)-(n^2+m^2)kt}$ where n and m are some constants. **(06)**

**c.** Let G be a non-abelian group of order $p^3$ where $p$ is prime then find $o(Z(G))$. **(02)**

**OR**

**Q-6**

**a.** Check whether the following polynomials are irreducible over $Q$ or not . **(06)**

i) $x^6 + x^3 + 1$

ii) $x^3 - 4x + 2$

iii) $x^{10} + x^9 + x^8 + x^7 + x^6 + x^5 + x^4 + x^3 + x^2 + x + 1$

**b.** Does the group of order 72 simple ? Justify and state the results you use. **(04)**

**c.** Using Euler's Modified method find $y(0.6)$ given **(04)**
$y' = 1 - 2xy\,, y(0) = 0$ .Take $h = 0.2$